Numerical study of SU(2) Yang-Mills theory with gluinos

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We report on a numerical investigation of the SU(2) gauge theory with gluinos. The low-lying spectrum in bosonic and fermionic channels is determined. Improvements of the multi-bosonic algorithm are discussed.

1. INTRODUCTION

Simulating supersymmetric field theory on the lattice is not straightforward. As the Poincaré symmetry is broken on the lattice, so is SUSY. A possible way out is to follow the ansatz by Curci and Veneziano [1]. They proposed to live with broken SUSY on the lattice and to reveal the relevant information in the continuum limit, where SUSY should be restored.

2. ACTION AND ALGORITHMS

The *Curci-Veneziano action* of $N = 1$ SYM, after integrating out the gluino field, is given by

$$S_{CV} = \beta \sum_{pl} \left(1 - \frac{1}{2}\mathrm{Tr}\, U_{pl}\right) - \frac{1}{2}\log\det Q[U] \ . \quad (1)$$

Here the fermion matrix is

$$Q_{yv,xu} = \delta_{yx}\delta_{vu} - K \sum_{\mu=\pm} \delta_{y,x+\hat{\mu}}(1+\gamma_\mu) V_{vu,x\mu} \quad (2)$$

and $V_{vu,x\mu}$ the gauge link variable in the adjoint (triplet) representation. The bare parameters are: the usual gauge coupling $\beta = 4g^{-2}$ and the hopping parameter of gluino K. The factor of $\frac{1}{2}$ in front of log det shows that the gluino is a Majorana fermion, corresponding effectively to a flavour number $N_f = \frac{1}{2}$. (For further discussion of the lattice action see [2].)

*Poster presented by K. Spanderen.

2.1. Two-step multi-bosonic algorithm

For the Monte Carlo simulations with dynamical gluinos we use the two-step variant [3] of the multi-bosonic algorithm [4]. This requires smaller storage and has shorter autocorrelations. For the correction step with the higher order polynomial P_n it turned out better to use another method, not the one introduced in ref. [3]. In terms of the hermitean fermion matrix $\tilde{Q} \equiv \gamma_5 Q = \tilde{Q}^\dagger$ let us define, with n even, the decomposition

$$\begin{aligned} P_n(\tilde{Q}^2) &= r_0 \prod_{j=1}^{n/2} (\tilde{Q} - \rho_j^*)(\tilde{Q} - \rho_j) \\ &\equiv P_{n/2}(\tilde{Q})^\dagger P_{n/2}(\tilde{Q}) \ . \end{aligned} \quad (3)$$

Using this form, the noise vector η, necessary in the *noisy correction* step, can be generated from the gaussian vector η' according to

$$\eta = P_{n/2}(\tilde{Q})^{-1}\eta' \ , \quad (4)$$

where $P_{n/2}(\tilde{Q})^{-1}$ can be obtained as

$$P_{n/2}(\tilde{Q})^{-1} = \frac{P_{n/2}(\tilde{Q})^\dagger}{P_n(\tilde{Q}^2)} \simeq P_{-n}(\tilde{Q}^2) P_{n/2}(\tilde{Q})^\dagger \ . \quad (5)$$

In the last step P_{-n} denotes a polynomial approximation for the inverse of P_n on the interval $[\epsilon, \lambda]$, which covers the spectrum of $\tilde{Q}^2 = Q^\dagger Q$ on typical gauge configurations. For the calculation of the necessary polynomials procedures written in Maple are available [5].

2.2. Preconditioning and eigenvalue distributions

In order to improve the performance of our fermion simulation algorithm, preconditioning according to ref. [6] turned out to be very useful. The hermitean fermion matrix is decomposed as

$$\tilde{Q} = \gamma_5 Q = \begin{pmatrix} \gamma_5 & -K\gamma_5 M_{oe} \\ -K\gamma_5 M_{eo} & \gamma_5 \end{pmatrix} \qquad (6)$$

and then we have

$$\det \tilde{Q} = \det \hat{Q}\,, \quad \text{with} \quad \hat{Q} \equiv \gamma_5 - K^2 \gamma_5 M_{oe} M_{eo}\,. \qquad (7)$$

The matrix $\hat{Q}^2$ has a smaller condition number λ/ϵ than $\tilde{Q}^2$. The condition number and its fluctuations on different gauge configurations are dominated by the minimal eigenvalue. A comparison of the fluctuations of the lowest eigenvalue of $\hat{Q}^2$ and $\tilde{Q}^2$ is shown in figure 1.

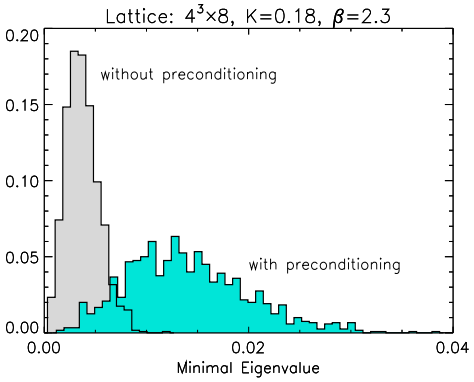

Figure 1. Distribution of the minimal eigenvalues of $\hat{Q}^2$ and $\tilde{Q}^2$ on $4^3 \cdot 8$ lattice at ($\beta = 2.3$, $K = 0.18$).

As a consequence of the smaller condition numbers and smaller orders of the polynomial approximations the autocorrelations are shorter with proconditioning than without it. For an example see table 1.

Random matrix models suggest that the fluctuations of the minimal eigenvalue are inversely proportional to the lattice volume. (For references and a recent summary see ref. [7].) This is advantageous for the choice of the interval of polynomial approximations $[\epsilon, \lambda]$. Our numerical data support the decrease of fluctuations, as is shown by figure 2.

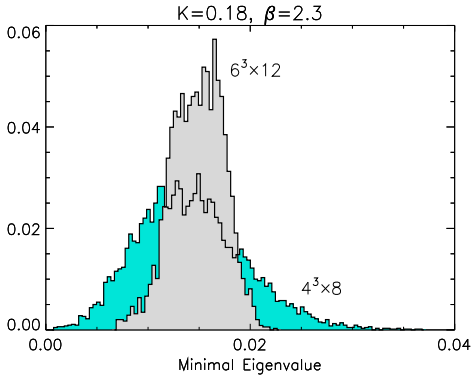

Figure 2. Distribution of minimal eigenvalues of the squared preconditioned hermitean fermion on $4^3 \cdot 8$ and $6^3 \cdot 12$ lattice at ($\beta = 2.3, K = 0.18$).

3. LOW-LYING SPECTRUM

A first important question for the numerical simulations is to determine the masses of lightest states in different channels as a function of the gluino mass. These can be compared to the predictions of the low energy effective action [8],[9]. It is expected that there is confinement as in pure gauge theory. In the limit of zero gluino mass the states should occur in degenerate supermultiplets.

The methods to determine low-lying masses can be tested and tuned on "quenched" gauge configurations generated with the pure gauge part of the action [10],[11].

3.1. Correlations

The lightest supermultiplet at zero gluino mass is presumably a massive chiral multiplet consisting of a scalar, a pseudoscalar and a spin-$\frac{1}{2}$ Majorana fermion. The bosonic states have analogues in QCD and can be made out of gluinos. Since the gluinos are in the adjoint representation, let

Table 1
Autocorrelation times for the plaquette value on $4^3 \cdot 8$ lattice at ($\beta = 2.3, K = 0.18$). δ^2 is the deviation norm of the polynomial approximation.

n	ε	δ^2	precond.	$\tau_{int}(\Box)$
16	0.0004	0.00085		194(38)
8	0.002	0.00052	×	65(17)

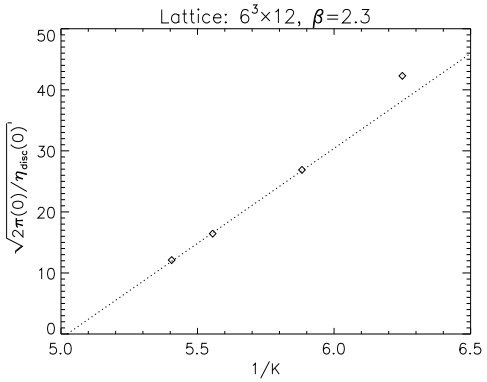

Figure 3. The square root of the ratio of the connected to disconnected part of the correlation for a-eta as a function of $1/K$. The line is a linear extrapolation through the last three points.

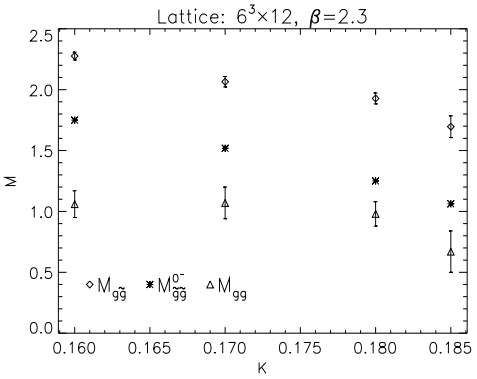

Figure 4. Dependence of the lowest bound state masses on the hopping parameter K on $6^3 \cdot 12$ lattices at $\beta = 2.3$.

us call them *a-eta* and *a-f_0* for pseudoscalar and scalar, respectively. The states made out of gluinos can be generally called *gluinoballs*. The two-fermion correlation functions contain connected and disconnected parts. The connected parts can be considered separately and, in analogy with QCD, can be associated with the *a-pion* and *a-sigma* for pseudoscalar and scalar, respectively. Near the *critical hopping parameter* K_{cr} corresponding to zero gluino mass the ratio of the connected to disconnected part has to be $\mathcal{O}(1)$. This is how the dependence of the chiral symmetry breaking pattern on the number of flavours can become manifest. As our numerical data obtained with the *volume source method* [12] show, for $K < K_{cr}$ the disconnected part is much smaller, but it is increasing for increasing K (see figure 3). Extrapolation to a ratio equal to one gives a rough estimate $K_{cr} \simeq 0.20$.

3.2. Masses

Besides the gluinoballs, confined states can also be composites of gluons and gluinos. Purely gluonic states are, as usual, called *glueballs* and the spin-$\frac{1}{2}$ fermionic states made out of gluinos and gluons as *gluino-glueballs*.

Our first series of Monte Carlo runs with dynamical gluinos has been performed at $\beta = 2.3$ and $0.16 \leq K \leq 0.185$ on $4^3 \cdot 8$ and $6^3 \cdot 12$ lattices. For the determination of the masses of glueballs and gluino-glueballs we used smeared sources [13]. Preliminary results on the masses of 0^+ glueball, a-eta and spin-$\frac{1}{2}$ gluino-glueball are shown in figure 4. As one can see, in this range of couplings the expected degeneracy of bound state masses is not yet observed. We are presently extending the runs to other parameter values and to larger lattices.

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